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Namo	e :				 	

Sixth Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2025 (2019 to 2022 Admissions) CORE COURSE IN MATHEMATICS 6B11MAT : Complex Analysis

Time: 3 Hours Max. Marks: 48

PART - A

Answer any 4 questions. Each question carries 1 mark.

- 1. Define a region in complex plane.
- 2. Evaluate $\oint_C e^z dz$ where C is the unit circle in counter clockwise direction.
- 3. State ML inequality for a complex function f(z).
- 4. What do you mean by singular point of a complex function?
- 5. Define residue of a function f(z) at a singular point z_0 .

 $(4 \times 1 = 4)$

PART - B

Answer **any 8** questions. **Each** question carries **2** marks.

- 6. What do you mean by complex continuous function? Give an example.
- 7. Show that $w = e^z$ is analytic everywhere.
- 8. Solve the equation $\cos z = 5$.
- 9. Evaluate $\oint_C \overline{z} dz$ where C is the unit circle in clockwise direction.



- 10. Evaluate $\oint_C \frac{\cos z}{(z-\pi i)^2} dz$ where C is the circle |z|=4 in counter clockwise direction.
- 11. Explain the idea of convergence of complex sequences. Give an example for a convergent complex sequence.
- 12. Prove that every absolutely convergent series is convergent.
- 13. Discuss the convergence of

$$\sum_{n=0}^{\infty} \frac{(100 + 75i)^n}{n!} \cdot$$

14. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty}\frac{(2n)!}{(n!)^2}(z-3i)^n\cdot$$

- 15. State Laurent's theorem for a complex function f(z).
- 16. Prove that the zeros of a non-zero analytic function f(z) are isolated. (8×2=16)

Answer any 4 questions. Each question carries 4 marks.

- 17. Prove that an analytic function whose absolute value is a constant, is a constant function.
- 18. Prove that the function w = In z analytic everywhere except at zero and on the negative real axis. Also find its derivative.
- 19. Evaluate $\int_C (z-z_0)^m dz$ where C is a circle with center at z_0 and radius r in counter clockwise direction.





20. If f(z) is analytic in a simply connected domain D, then prove that the integral of f(z) is independent of path in D.

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- 21. Discuss the convergence of geometric series.
- 22. Find all the power series expansions of $\frac{1}{1-z}$ with center 0.
- 23. Integrate $f(z) = \frac{\sin z}{z^4}$ counter clockwise around the unit circle. (4×4=16)

Answer any 2 questions. Each question carries 6 marks.

- 24. State and prove the necessary condition for a function f(z) = u(x, y) + iv(x, y) to be analytic at a point z_0 .
- 25. State and prove Cauchy's integral formula.
- 26. Find the Taylor's series for the function $f(z) = \frac{2z^2 + 9z + 5}{z^3 + z^2 8z 12}$ at the point z = 1.
- 27. Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0. (2×6=12)